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AUTHOR Bassler, Otto C.; And Others
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ABSTRACT

Two distinct strategies for teaching the solution to verbal problems were compared. Programs of instruction were prepared that reflected the Polya Method (read and understand the problem, plan for a solution, carry out the plan, and check the result) and the Dahmus Method (translate the verbal statements into mathematical symbols prior to solving the problem and checking the solution). A form of individualized instruction was used to present these programs to 53 ninth-grade algebra students classified into three ability levels based upon Algebra Prognosis Test scores. Following seven days of instruction, data were gathered on an investigator-constructed posttest and retention test which were scored on an equation criterion and a problem solution criterion. Results showed that Polya Method students scored higher than Dahmus Method students on the equation criterion but there was no difference between the two groups on the problem solving criterion, that scores on both criteria are highly resistant to forgetting, and that ability level differences occurred. (Author/DT)

Final Report





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Otto C. Bassler
Morris I. Beers
Lloyd I. Richardson
George Peabody College for Teachers
Box 38
Nashville, Tennessee 37203

# COMPARISON OF TWO INSTRUCTIONAL STRATEGIES FOR TEACHING THE SOLUTION TO VERBAL PROBLEMS

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U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE

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Abstract

Two distinct strategies for teaching the solution to verbal problems were compared. In one strategy, the Polya Method, the students were taught to read and understand the problem, plan for a solution, carry out the plan and check the obtained solution. The second strategy, the Dahmus Method, is based upon a direct, pure, piece-meal and complete translation of the verbal statements into mathematical symbol statements prior to solving the problem and checking the solution.

Programs of instruction were prepared to reflect the two treatments. A form of individualized instruction was used to present these programs to 53 ninth-grade algebra students who were classified into three ability levels based upon Algebra Prognosis Test scores. Following seven days of instruction data were gathered on an investigator constructed posttest and retention test which were scored on an equation criterion and a problem solution criterion.

The results showed that (1) Polya Method students scored higher than Dahmus Method students on the equation criterion but there was no difference between the means of these groups on the problem solution criterion; (2) the scores on both criteria are highly resistant to forgetting; and (3) ability level differences manifested themselves as expected.

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# Comparison of Two Instructional Strategies for Teaching the Solution to Verbal Problems

Otto C. Bassler
Morris I. Beers
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## Comparison of Two Instructional Strategies for Teaching the Solution of Verbal Problems

### Introduction

Problem solving occupies a central role in the teaching and learning of mathematics. One of the techniques used for teaching problem solving is to teach the solution of verbal problems which begins quite early in the mathematics curriculum with rather simple problems designed to permit students to gain maturity in thinking and progresses to complex problems. Verbal problem solving causes many students difficulty. Manheim (1961) states that "the 'word problem' remains a generator of fear and frustration for many students. ...If we ask our students, or ourselves, why such a problem is more difficult than a non-word problem, we are apt to find the difficulty attributed to 'the nonmathematical nature of the problem.' (p. 234)" The nonmathematical nature of these problems is that they are stated in the English language, which requires reading and understanding what has been read, rather than being stated in mathematical symbolism. The reading of English statements is thought to be critical by Barney (1972) when he stated that "some children read mathematics narrative problems so slowly and laboriously that before they come to the words at the end of the sentence they have forgotten those at the beginning. But when the child does finish reading the problem, he is faced with the additional task of structuring, editing, and rearranging the mathematical concepts in order to produce a meaningful answer. (p. 131)"

One major reason for the apparent difficulty in the solution of verbal problems is that students do not have a good method for attacking and solving them. Breslich (1920) felt that "with a technique directing thinking similar to skills work, the solution of verbal problems may be made a matter as simple, or even more simple than most of the skills work. (p.2)" The implication here is that teachers should have a major impact upon the student's ability to solve verbal problems by identifying effective strategies for teaching verbal problems and then using these strategies to teach students to solve verbal problems.

Unfortunately, Breslich's suggestion has not had a major impact on methods used to teach the solution of verbal problems in the classroom. Stilwell (1967) found that a relatively small amount of class time was devoted to discussing a method for solving problems. As a matter of fact, less than three per cent of the time spent in problem solving activity was devoted to developing a method for solving problems. He also found that only seven per cent of the problem solving activity was devoted to looking back at a problem or ahead to its implication. Relating these statements to the solution of verbal problems, it would seem that teachers avoid teaching a general strategy which may apply to many types of verbal problems,

but rather attempt to identify an algorithm which is limited in scope and applies to only one type of problem. This is also true of several current textbooks which restrict each section of a chapter entitled, "verbal problems" to a specific type of problem. Due to these and other difficulties, many teachers avoid presentation of units dealing with verbal problems because of the lack of a well defined instructional technique. These remarks confirm the convictions of Van Engen (1963) who believed that there was little emphasis placed upon the solution of verbal problems.

## Statement of the Problem

This investigation is designed to assess the relative effects of two distinct strategies of instruction when students are learning to solve verbal problems. The first strategy is derived from the work of Polya (1957) and presented in the book, How to Solve It. This strategy is basically heuristic in nature; that is, the student is expected to read and understand the problem; to plan for a solution of the problem, which includes identifying the unknown, identifying relations and operations to be performed, and drawing analogies to similar problems which may have been previously solved; to carry out the plan; and, finally, to examine the obtained solution. This strategy will be referred to as the Polya Method (PM). The second strategy is based upon the work of Dahmus (1970). He refers to his method as the DPPC Method, the letters of which stand for the words, direct, pure, piece-meal, and complete. In essence, this method is based upon a direct, pure, piece-meal and complete translation of the verbal statement into mathematical symbolism. Following translation, the mathematical symbols are used to find the solution for the problem. This second strategy will be called the Dahmus Method (DM). The two methods appear to be diametrically opposed, in that, PM encourages the student to look at the problem as a whole prior to beginning work while DM encourages the student to translate information as it appears in the problem. Both of the procedures, however, have been described by their respective originators as the method to solve verbal problems.

Since DM emphasizes a translation from verbal statements to mathematical symbol statements, it is hypothesized that DM subjects will achieve significantly higher translation scores following instruction. Translation scores will be based upon the ability of subjects to write a mathematical equation for the problem prior to its solution. The PM method emphasizes rational solutions and for this reason it is hypothesized that PM subjects will score significantly higher than DM subjects when only the solution to verbal problems is assessed. Each of these variables will be investigated at three ability levels. Since there is no reason to suspect that these treatments will produce different results for different ability levels, no interaction between ability and treatment are hypothesized.

## Review of the Literature

There has been a great deal of research done in the problem solving area. One set of research studies deals with factors that are influential

in the problem solving process. Factors considered in this research include many aspects of reading, computational abilities and intelligence. Alexander (1960), Chase (1960) and Trueblood (1969) all reported that reading skills were related to success in problem solving. Banghart and Spraker (1963) and Suydam and Weaver (1970) found intelligence highly correlated with problem solving ability. There have, however, been other studies notably Henney (1968) and Balow (1964) which indicate that IQ and general reading ability are not highly correlated with problem solving ability.

A second set of studies investigate the influence that teachers have on students' ability to solve problems. In studying the effect of teacher verbal behavior on pupil achievement in problem solving in the sixth grade, Lamanna (1968) classified teachers in terms of their verbal behavior in the classroom. There was some indication from this study that verbal behavior on the part of the teacher did increase the achievement scores of the above average student. The teacher's discouragement of the student talk resulted in increased achievement on the part of average students. Godgart (1965) who investigated teacher characteristics pertaining to problem solving concluded that there is no relationship between a teacher's problem solving ability and his effectiveness in teaching problem solving and there is no apparent relationship between the number of course hours in mathematics and problem solving ability on the part of the teacher.

A third set of studies and those most relevant to the present investigation are those which compare different strategies for teaching the solution to verbal problems. The number of studies in this area is sparse but there has been some pioneering work done. One of the first studies to test the relative effectiveness of two strategies for teaching verbal problems was conducted by Clark and Vincent (1925). In this study, the step method was compared with the graphical method for solving word problems. In the step method the students were directed to read the problem, select what was wanted, determine what facts were given, find necessary relationships and operations for the correct solution and then to solve the problem. The graphical method directs the pupil to determine what is to be found in the problem, what it depends on, what each of these dependents in turn depend upon, and so on, until the essential facts and relationships in the problem have been identified. These facts and relations are then expressed as a type of schematic diagram which indicates a graphical solution to the problem. In testing the relative effectiveness of these two strategies, Clark and Vincent found that use of the graphical method is preferable when assessment is made immediately following instruction, while the step method appears to be preferable when longer term effects are assessed.

Another study was conducted by Kinsella (1951) who tested the step approach against the use of no formal teaching method. His findings were that the student's success in selecting the correct process was not dependent on the prior success with any certain step or combination of steps, and that the step method might lower the level of performance on the solution of the whole problem. This suggests that formal analysis which requires pupils to answer a set of questions in solving each problem may not produce superior results.

Wilson (1967) tested three approaches of teaching the solution to verbal problems to elementary school children. One approach was the wanted-given method where the student was to find the wanted from the given. The second approach was the action sequence method where the student obtained the necessary operation by recognizing the action that occurred in the problem situation. The third approach used as a control group allowed students to solve problems in any manner with no specific type of instruction. The test results in this study favored the wanted-given approach.

The search of the literature did not reveal any studies which compared the step approach with a translation approach. While there does not seem to be any hard data comparing this method with others, there is a great deal of opinion. Dahmus (1970) who originated the DPPC translation method, claims that the translation process serves to reduce all verbal problems to about the same level of difficulty. He says that he has used this method with success in a variety of different classes. May (1968) reported success with direct translation techniques at the elementary level and Schoenherr (1968) had success with a variation of this direct translation method in helping students write a single equation to represent work they had just completed in solving word problems.

Cohen and Johnson (1967) claim that when a student acquires the ability to translate accurately from a certain description of a physical situation to an appropriate mathematical sentence, it will allow him to cope with a large number of problems in an orderly and logical fashion. Apparently this ability to translate any situation into some form of mathematical symbolism is the most useful tool in problem solving. Manheim (1961) concurs with this issue when he claims that the reason for transforming expressions from the non-mathematical language to mathematical language is to have greater manipulative facility.

## Method

### Instructional Materials

A program of instruction was devised to teach students how to solve verbal problems. On the first day of instruction, the students were instructed in the selection of variables which could represent unknown values in a problem. This instruction was common to both treatments. Following this common instruction, students were introduced to the strategy that they would be using in solution of verbal problems. The second day of instruction reviewed the notions of variable and instructed students in the solution of simple linear equations in one unknown. The equations that were considered were of the form $a + x = b$, or $ax + b = c$ where a, b, and c have specific numerical values and x is the variable to be found. The two topics, variable and solution of simple linear equations were the only instruction common to both treatments. The remainder of the instructional time was spent in the development of the particular strategy to be used in the solution of verbal problems and guided practice in applying this strategy to the solution of verbal problems. The amount of guidance decreased as students progressed through the instructional sequence. Immediate feedback pertaining to the correctness of response was provided as students were completing the instructional materials.

In order to reduce variability due to the teacher variable, a form of individualized instruction was used. The instructional materials were prepared as a sight-sound presentation. The sight-sound unit for each presentation was comprised of a series of colored slides which portrayed sample problems photographed at various stages of completion and questions about these problems were then projected onto a screen for students to view. This slide presentation was synchronized with a tape program which provided instruction and directed the learning process. The function of the tape presentation was to inform the learner of the task to be performed, read and explain materials presented on the slide, explain in detail the steps in the particular strategy of instruction, direct the student to complete various tasks or problems, and following completion, to inform the students of the correctness of their response. When students were directed to complete a problem or to perform individual work the tape player was stopped but the slide display remained visible for the student to use.

Two different sight-sound presentations were constructed by the investigators to reflect the two different teaching strategies, PM and DM. These were prepared to reflect the treatment differences explained in the following paragraphs.

Polya Method. PM directed students to complete the solution of verbal problems by carrying through the six steps listed below.

1. Read the problem carefully. Students were directed to read the problem along with the tape and then were instructed to reread the problem as many times as necessary to gain understanding of the problem and its conditions.

2. Decide what question the problem asks and choose a variable to represent the unknown. Here, the students were directed to reread the problem with the specific task of identifying the value or values to be found and to represent these by a variable or variables.

3. Consider the other information given in the problem and how it relates to the unknown. Again, students were asked to reread the problem and to identify relations between the given information and the wanted information.

4. Write an equation or equations expressing the given relationships.

5. Solve the equation or equations.

6. Check your answer. Incorporated in this step were suggestions to first check the reasonableness of an answer and then to check this solution in the original verbal problem.

An example of one of the first problems solved using this method is included in Appendix A. A great deal of guidance is provided for the student in completion of this exercise.

Dahmus Method. DM is a more structured method than PM. In this method the student is directed to translate each phrase as it appears in the verbal problem. For this reason, original instruction had to be given in translation of key phrases such as "increased by" is translated as "+"; "decreased by" is translated as "-." Students were directed to first translate all phrases of the word problem to mathematical statements. Each value to be found in the problem was translated as variable equals question mark. After all of the verbal statements were translated to mathematical symbol sentences, students were directed to find relations expressing the values to be found and to solve the resulting equation. This method used somewhat more slides than PM, but time was roughly equal in both of the treatments. An example of this treatment is provided in Appendix B, where the same problem, as illustrated for PM, is illustrated using DM, the direct, pure, piece-meal and complete translation method.

Each of the programs was designed to consume seven forty-minute instructional periods. The time for each program was roughly comparable. There was somewhat more tape time and more slides used by DM, but there were more pauses and time for student work using PM. The seventh day of this instructional sequence was devoted to practice problems. After reading the problem, students were directed to complete all steps in each of the methods before the tapes provided any feedback about the solution of the problem. Problems were explained on this day, but it was after the student had completed individual work rather than before.

## Tests

The Orleans-Hanna Algebra Prognosis Test. This test is designed to predict achievement in first year algebra. It incorporates questionnaire items to assess interest and personality characteristics, and a work sample approach to assess mathematical ability and proficiency. The questionnaire contributes a maximum of 40 points to the total test score and the work sample contributes at most 58 points.

A longitudinal study, extending over a full year of algebra, was conducted to assess predictive validity. The results of this study yielded correlation coefficients of test scores with algebra achievement data ranging from .72 to .83. These are claimed to establish a high level of predictive validity. The test-retest reliability coefficient on the total score was reported as .95. Thus, for the purpose of this study, it was assumed that the total raw score achieved on this instrument was a consistent and valid indicator of a students ability in Algebra I.

The Posttest. The investigator constructed posttest consisted of ten verbal problems. Seven of these problems were similar in nature to problems considered by the learners in the instructional phase of the study. The remaining three problems were different and more complex than the preceding seven. A copy of this test is included in Appendix C.

Students were directed to first find an equation for each verbal problem and then to solve the verbal problem. It was stressed that even if they could not find an equation, they should attempt to find the solution. This procedure yielded two different scores for each student; that is, a translation score which was defined to be the number of problems for which the student wrote the correct equation and a solution score which was the number of verbal problems for which the student had the correct answer.

Following administration of this test, an estimate of reliability was determined by computing Cronbach's alpha. For the translation score, alpha was determined to be .84; for the solution score alpha was .78. Based upon these values, it was assumed that the criterion measure possessed adequate reliability for comparison of cell means.

The Retention Test. The retention test also consisted of ten verbal problems. The items on this test were identical in content to those on the posttest; however, numerical values and context of the problems were changed to prevent direct recall of solutions. A copy of the retention test is included in Appendix D.

## Subjects

The learners employed in the experiment were 53 ninth-grade algebra students at St. Cecilia Academy, a private, catholic girl's secondary school, in Nashville, Tennessee. These students were in three classes, two of which were in the academic stream and the third in a basic mathematics stream. Although these learners probably represent an above average population, there was a wide range of ability as indicated on the Algebra Prognosis Test. This test is a part of the school's regular testing program and all ninth-grade students had completed it at the beginning of the school year.

Students were first subdivided into three ability groups on the basis of their Algebra Prognosis Test scores. Within each ability level, equal numbers of learners were assigned at random to one of the two treatment groups. Three of these students were eliminated from the study due to excessive absences and two more were randomly eliminated to have an equal number of subjects per cell. Data analysis was carried out on a total n of 48, with 8 students in each of the six cells.

Previous instruction in the Algebra I course had consisted of a review of sets and set terminology, an introduction to the terminology of algebra (variable, domain, range, algebraic expression), and graphing sets of points on the number line. There had been no formal instruction in Algebra I dealing with the translation of verbal sentences to algebraic sentences nor with the solution of verbal problems.

## Procedures

After the initial machine and the instructional materials had been developed, a pilot study was conducted at Overbrook School using 10 seventh and eighth-grade subjects. This pilot study pointed to a number of flaws in the instructional materials which were corrected prior to the final

development of the learning materials. In addition, the pilot study yielded the information that seventh and eighth-grade subjects lacked familiarity with variables and open sentences that were necessary for successful completion of this program.

The major revisions suggested from the pilot study and incorporated into the main study were:

1. To provide additional instruction in the identification and use of variables.

2. To provide instruction in the solution of open sentences.

3. To reduce the number of problems that the subjects were to complete each day.

4. To use beginning ninth-grade algebra students as subjects in the main study.

In the main study, students from three classes of Algebra 1 were assigned at random to one of the two treatment groups. Since two of these classes met at the same time, all of the PM subjects were reassigned to one room while all of the DM subjects were assigned to another room. Within each of these groups, the students were randomly assigned to one of the three machines which were used to present each treatment. This resulted in groups of 8 students receiving instruction from one of the sight-sound units. In order to further individualize instruction each student received the taped portion of the lesson through headsets while they were watching the slide presentation.

On the first day of the study, the students were instructed in the use of the machine. They were directed to listen carefully to the taped presentation while they were watching the slides. Further, they were told that at times they would be told to perform certain computations or answer certain questions on their own. When this occurred they were directed to respond to the question and then to raise their hand. After each student had completed the response individually, an investigator would restart the tape machine and go on to the next slide. In this way, each student had sufficient time to complete all of the assigned tasks in the prescribed manner. By grouping students on the PM and DM machines according to ability, the time needed for students to complete the exercises on each machine were roughly comparable and no student had to wait an excessively long time for the rest of the students to complete any exercise.

The instructional phase of the experiment consisted of seven 40-minute periods. On each of the instructional days, the students reported to the same machine in the same room to which they had been assigned. On the school day following the seven instructional days, the posttest was administered. This was a paper-pencil test and was administered in group fashion to all of the subjects. Approximately four weeks later, a retention test, a parallel form of the posttest, was administered to all subjects. The retention test was unannounced so that students did not have the opportunity to review for this test.

Since there was the possibility that different scorers would rate test items in a different fashion, two of the investigators scored each of the tests independently. On the equation criterion, each investigator scored the item as correct or incorrect with no part-credit. This same procedure was followed on the solution criterion. The two independent ratings were then summed to yield the equations score and the solutions score for each individual. Using this scoring technique, the maximum score for either of the two measures was 20.

## Experimental Design

The design used to analyze the data was a three-factor analysis of variance with repeated measures on one factor. The factors were ability (high, intermediate, and low), treatment (Polya, Dahmus) and test occasion (post, retention). This basic design was used for both of the criteria tested. A schematic diagram is provided in Figure I.

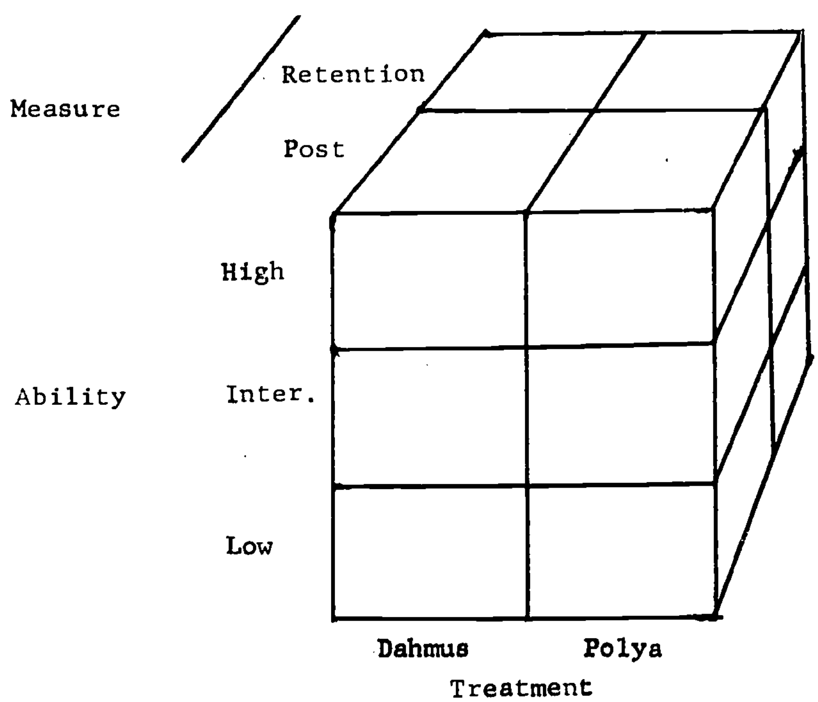


Figure I. The Experimental Design

By utilizing this design, the following null hypotheses are to be tested for each criterion measure.

1. There are no significant interactions of any order.

2. There are no s[illegible]rences in the posttest mean and the [illegible] mean.

3. There are no significant differences in the ability level means.

4. There are no significant differences in the treatment means.

Background information for this type of design and its analysis can be found in Winer (1962, pp. 337-349).

## Results

<u>Algebra Prognosis Test</u> scores were used to separate the subjects into high, intermediate, and low levels of algebraic ability, the second factor in the design. Means and standard deviations of these scores for each cell are shown in Table 1.

Table 1

Means ($\overline{X}$) and Standard Deviations (s.d.) of Scores on the Algebra Prognosis Test for Each Cell

| | Treatment | | | |
|---|---|---|---|---|
| | Polya | | Dahmus | |
| Ability Level | $\overline{X}$ | s.d. | $\overline{X}$ | s.d. |
| High | 83.5 | 1.9 | 85.9 | 2.4 |
| Intermediate | 75.4 | 3.6 | 72.4 | 1.2 |
| Low | 60.1 | 13.5 | 63.8 | 5.0 |

(n per cell = 8)

A casual observation of this table indicates that the means for each ability level are similar across treatments. The difference between means was not tested since subjects were randomly assigned to treatment within each ability group. Examination of this table, however, confirms the assumption that randomization provided comparable groups.

Separate analyses were made on the equation criterion and the problem solution criterion. Means and standard deviations of the scores on the

equation criterion of the posttest and the retention test are shown in Table 2.

Table 2

Means ($\bar{X}$) and Standard Deviations (s.d.) of Scores on the Equation Criterion of the Posttest and Retention Test for Each Cell

| | Treatment | | | |
|---|---|---|---|---|
| | Polya | | Dahmus | |
| Ability Level | $\bar{X}$ | s.d. | $\bar{X}$ | s.d. |
| Posttest | | | | |
| High | 12.5 | 2.6 | 9.6 | 5.6 |
| Intermediate | 8.3 | 5.6 | 2.1 | 3.1 |
| Low | 4.9 | 2.3 | 1.4 | 1.2 |
| Retention Test | | | | |
| High | 13.1 | 5.0 | 8.6 | 6.3 |
| Intermediate | 9.6 | 4.9 | 1.9 | 1.6 |
| Low | 3.9 | 3.0 | .4 | .7 |

(n per cell = 8)

Graphs of the mean scores which are reported here are pictured in Figure II. These graphs show a definite and expected ordering of the means with regard to ability. It can also be seen that there appears to be a treatment effect since the graphs of the PM means are completely above the graphs of the DM means.

To determine the significance of treatment effects, ability effects, or interaction effects, an analysis of variance was performed on the data reported in Table 2. The summary table for this analysis of variance is shown in Table 3. All hypotheses were tested at the .05 level of significance.

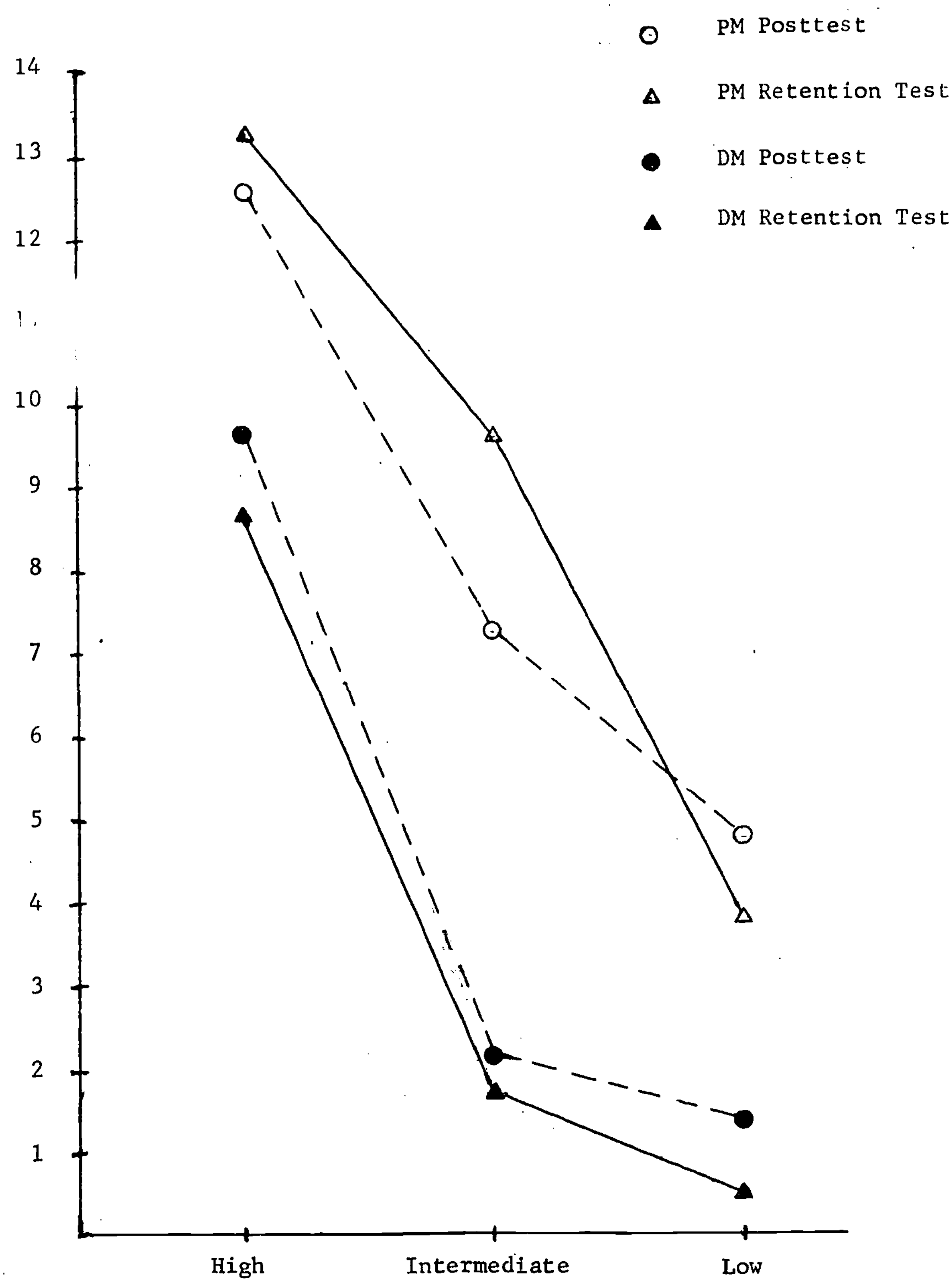


Figure II. Mean Scores on the Equation Criterion of the Posttest and Retention Test for Treatments Within Ability Levels.

Table 3

Summary Table for the Analysis of Variance of

Scores on the Equation Criterion

| Source of Variation | Degrees of Freedom | Mean Square | Variance Ratio |
|---|---|---|---|
| Between Subjects | 47 | | |
| A (Strategy) | 1 | 532.04 | 20.30* |
| B (Ability) | 2 | 575.76 | 21.97* |
| A X B | 2 | 29.88 | 1.14 |
| Error Between | 42 | 26.20 | |
| Within Subjects | 48 | | |
| C (Testing Occasion) | 1 | 1.04 | 0.21 |
| A X C | 1 | 7.04 | 1.45 |
| B X C | 2 | 4.88 | 1.00 |
| A X B X C | 2 | 1.76 | 0.36 |
| Error Within | 42 | 4.87 | |
| Total | 95 | | |

*Indicates $P < .05$.

The results of this analysis of variance were as follows:

1. There were no significant interactions of any order. This then permits interpretable tests of the main effects.

2. The effect of treatment was significant. Comparing this with the results reported in Table 2 indicates that the PM subjects had a significantly larger mean score than the DM subjects on the equation criterion.

3. As was expected, there was a significant ability effect. Although individual comparisons were not made, it appears

that the mean score of the high ability subjects was greater than that for the intermediate ability subjects which, in turn, was greater than that for the low ability subjects.

4. There was no significant difference between the mean posttest score and the mean retention test score.

Means and standard deviations of scores on the problem solution criterion of the posttest and retention test are reported in Table 4.

Table 4

Means ($\bar{X}$) and Standard Deviations (s.d.) of Scores on the Problem Solution Criterion of the Posttest and Retention Test for Each Cell

| | Treatment | | | |
|---|---|---|---|---|
| | Polya | | Dahmus | |
| Ability Level | $\bar{X}$ | s.d. | $\bar{X}$ | s.d. |
| Posttest | | | | |
| High | 12.5 | 4.0 | 12.5 | 4.8 |
| Intermediate | 9.5 | 4.9 | 7.6 | 3.9 |
| Low | 6.8 | 4.9 | 5.0 | 3.2 |
| Retention Test | | | | |
| High | 14.4 | 4.3 | 12.0 | 4.3 |
| Intermediate | 9.0 | 4.5 | 9.3 | 5.4 |
| Low | 8.6 | 4.4 | 7.1 | 3.4 |

(n per cell = 8)

It is interesting to note that the means on the problem solution criterion are greater than those obtained on the equation criterion. This was true across groups, but it was dramatically so for the low and intermediate ability subjects. The means that are reported in Table 4 are graphed in Figure I. As before, there appears to be a marked ability effect. It appears, however, that the effect due to treatment is not as great for this criterion as it was for the equation criterion. It is also surprising to note the high level of retention that is illustrated in this graph.

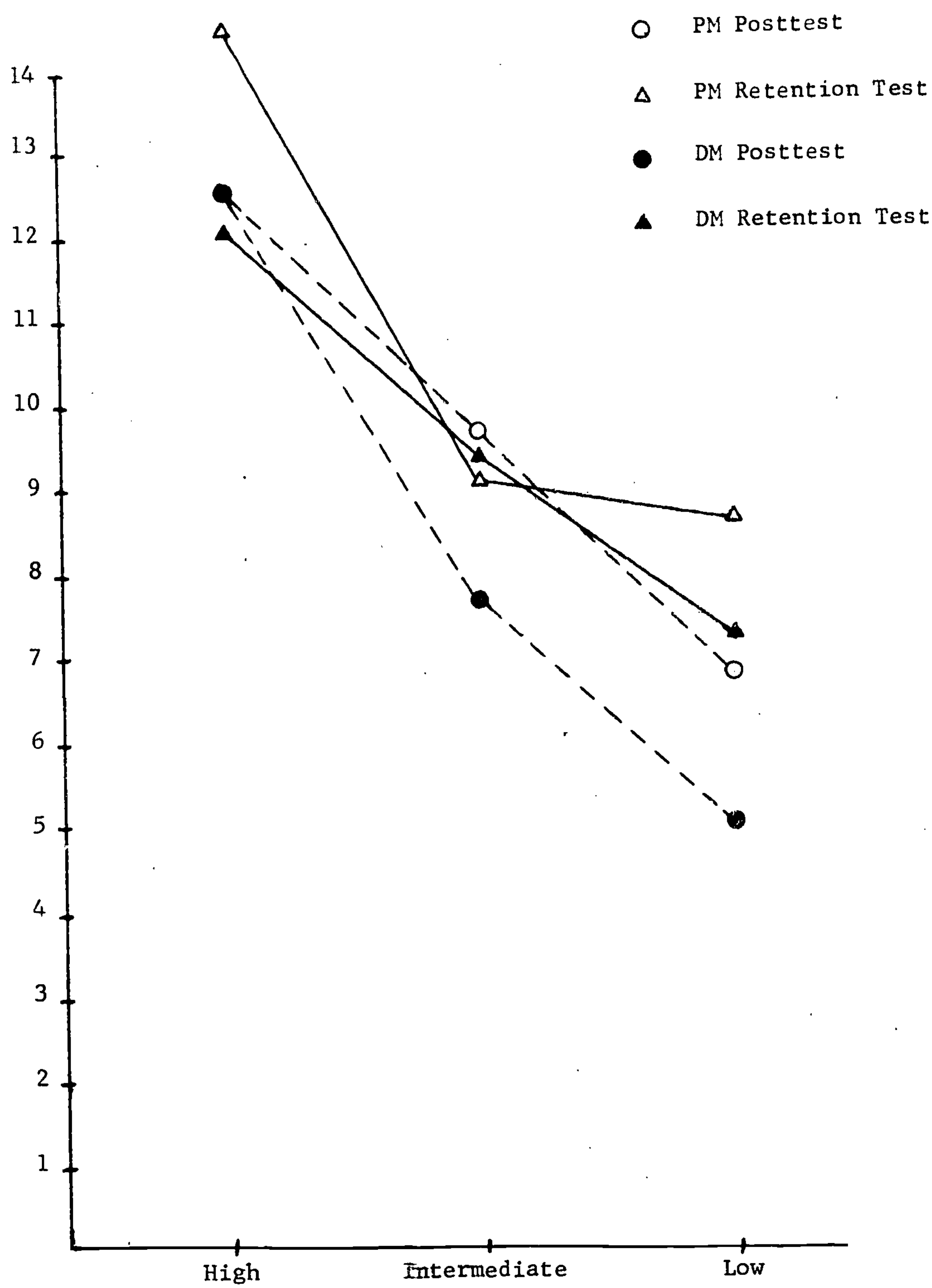


Figure III. Mean Scores on the Problem Solution Criterion of the Posttest and Retention Test for Treatments Within Ability Levels.

To determine if the effects of treatment, ability, or test occasion were significant, an analysis of variance was performed on these data. The summary table for this analysis is reported in Table 5.

Table 5

Summary Table for the Analysis of Variance of

Scores on the Problem Solution Criterion

| Source of Variation | Degrees of Freedom | Mean Square | Variance Ratio |
|---|---|---|---|
| Between Subjects | 47 | | |
| A (Strategy) | 1 | 35.04 | 1.07 |
| B (Ability) | 2 | 296.01 | 9.03* |
| A X B | 2 | 1.32 | 0.04 |
| Error Between | 42 | 32.79 | |
| Within Subjects | 48 | | |
| C (Testing Occasion) | 1 | 28.16 | 5.00* |
| A X C | 1 | 0.00 | 0.00 |
| B X C | 2 | 5.07 | 0.90 |
| A X B X C | 2 | 10.21 | 1.82 |
| Error Within | 42 | 5.62 | |
| Total | 95 | | |

*Indicates $P < .05$.

The results of this analysis were as follows:

1. There were no significant interaction effects of any order. As before, this permitted an interpretable test of the main effects.

2. There was no significant effect due to treatment.

3. There was a significant ability effect. Comparing this with the data presented in Table 4, it appears that the

mean for the high ability subjects is greater than that for the middle ability subjects which, in turn, is greater than that for the low ability subjects.

4. There was a significant test occasion effect. Looking at the means, it appears that the mean for the retention test is significantly larger than the mean for the post-test.

In summary, the results showed that (1) PM groups scored higher than DM groups on the equation criterion, but there was no difference between the means of these groups on the problem solution criterion; (2) The scores on both criteria are highly resistant to forgetting. There was no significant difference between the test occasion means on the equation criterion and the retention test mean score was higher than the posttest mean score for the problem solution criterion; (3) Ability level differences manifested themselves as expected; that is, high ability subjects scored better than did intermediate ability subjects who, in turn, scored better than low ability subjects. There were no interactions between any combination of treatment, ability level and test occasion.

## Conclusions

From the results, it appears that when ninth grade algebra students are taught to solve verbal problems using teaching machines as the sole source of instruction, the Polya method is somewhat preferable to the Dahmus method. The basis upon which this conclusion is drawn is the result that PM subjects had a significantly larger mean score on the equation criteria and performed at least as well on the problem solution criteria. There are, however, a number of limiting conditions which lead the investigators to accept this conclusion with caution.

In a general sense, the performance of all of the subjects on the post learning measures was not outstanding. Referring to Tables 2 and 4, it can be seen that the mean attainment for the cells in percentages ranged from 2% to 65% on the equation criterion and from 25% to 75% on the problem solution criterion. This implies that the level of attainment for the subjects was not as high as had been hoped by the investigators. This lack of learning would more than likely suppress differences between groups of subjects.

Perhaps one problem with research of this type is that in an attempt to control for the teacher-variable, teaching machines were employed as the sole source of instruction. This method does provide for some good teaching techniques; for example, a carefully sequenced set of instructions and immediate confirmation of response. It does not, however, permit the students to ask questions or clear up any misunderstandings that are peculiar to that student. Perhaps since learning how to solve verbal problems is a difficult task, it is necessary for the student to have more interaction with teachers or other pupils.

A second condition that may have been operating in this experimental study is the history of the subjects themselves. Solution of verbal problems is not new to students in ninth grade algebra. In fact, many verbal problems have been solved by these students in the seventh and eighth grade mathematics curriculum. From examinations of seventh and eighth grade textbooks, it seems that if the textbook presents a general strategy for solving verbal problems, the strategy is based upon a step method similar to that defined as PM in this study. Thus, it may be that PM subjects had training which reinforced previous problem solving techniques and DM subjects had to learn a new strategy. This may have inhibited learning in at least the initial stages of the solution to verbal problems for the DM group.

Finally, it should be emphasized that DM subjects did learn to translate from verbal sentences to mathematical symbol sentences. In analyzing the test papers, it was clear that a large majority of the DM subjects translated directly from the verbal statements to mathematical symbol statements. They did not, however, combine these mathematical symbol statements into one acceptable equation which could be used to find the solution to the problem. It appears that they did use their translated information to find the solution to the problem without obtaining a general overall equation, since there was no significant difference between the groups on the problem solution criterion. Failure to reject the null hypotheses on the problem solution criterion for the two treatments would suggest that any differences in this assessment are due to chance factors and the two groups performed equally well.

It is also concluded that solution to verbal problems once learned is resistant to forgetting. It is recognized that the subjects in this study continued in the Algebra I class which is a problem solving type course. The investigators were assured, however, that no direct instruction pertaining to the solution of verbal problems had occurred in any of these classes between the administration of the posttest and the retention test.

The result indicating a higher mean score on the retention problem solution criterion was somewhat surprising. To further investigate this finding, two subscales for each measure were obtained. The first subscale consisted of the seven problems which were similar in nature to problems considered during instruction. The second subscale consisted of the three problems which were more complex than those considered during instruction. It was interesting to note that on the equation criterion, the analysis of variance tables for the subscales were almost identical to those obtained for the total equation scores. On the problem solution criterion, however, there was no difference between testing occasions for the seven problem subscale. There was a significant difference on the three problem subscale; that is, the scores on the retention test were significantly larger than the scores on the posttest. Hence, it appears that the subjects made gains in solving problems which were more complex than those on which they had received instruction.

In conclusion, it should be emphasized that the implications of this study for teaching the solution to verbal problems are only tentative. It does seem that both of these methods can be used for teaching the solution

to verbal problems. If any generalizations are to be made, it is recommended that students receive instruction similar in nature to that described here as the Polya method. Perhaps for teaching in the classroom a synthesis of the two strategies of instruction could be used. The pure strategies, if they may be called that, could then be reserved as alternative strategies for students who are having difficulty in using the synthesized strategy.

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APPENDIX A

INSTRUCTIONAL SEQUENCE FOR SOLVING A PROBLEM

USING THE POLYA METHOD

Frame: 2:24

JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

Here's a problem you can do. /

Please read the problem with me. / (Read the problem.)

Now read the problem again and raise your hand when you're done. /**/

Frame: 2:25

JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

Now that you have completed step 1 in our method for solving word problems, go on to step 2. / Decide what question the problem asks. /4/ Write the question asked by the problem in Box 6 of your work sheet, <u>then</u> choose a variable to represent the unknown and write it in the same box. /**/

Frame: 2:26

HOW MUCH FARTHER WOULD JUDY HAVE TO GO TO

GET TO SCHOOL?

Voice:

The problem asks you to find how much farther Judy would have to go to get to school. / You are asked to find a distance. /*/

Frame:

H FARTHER WOULD JUDY HAVE TO GO TO

GET T OOL?

LE ISTANCE SHE MUST GO = D

Voice:

the variable to represent that distance. /

You ld have chosen some other variable, but we will be using for the rest of the problem.

If you wrote a different letter in Box 6, change it to a D now. /*/

Frame: 2:28

SCHOOL IS 7.[illegible] MILES FROM HER HOUSE. IF SHE

2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER

WOULD SHE HAVE TO GO TO GET TO SCHOOL.

Voice:

Let's go on to step 3. / What other information is given in the problem? How does this relate to the unknown? /

In Box 7 of your work sheet, write as many short statements as you need to express all the remaining information given in the problem. /**/

Frame:

1. JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE.

2. SHE RODE 2.7 MILES ON HER BICYCLE.

Voice:

The problem tells you that Judy's school is 7.3 miles from her house and that she rode 2.7 miles, of that distance, on her bicycle. /*/

Frame: 2:30

1) JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE.

2) SHE RODE 2.7 MILES ON HER BICYCLE.

D = THE DISTANCE JUDY MUST GO

Voice:

Using the two pieces of information, 1) and 2), along with the variable, D, you selected to represent the distance Judy still has to go, find a relationship which combines the information in these sentences. /

Write an equation using this relationship in Box 8 on your work sheet. /**/

Frame: 2:31

$D = 7.3 - 2.7$

OR

$D + 2.7 = 7.3$

Voice:

You could have written either $D = 7.3 - 2.7$ or $D + 2.7 = 7.3$. /

Since these two equations are equivalent it doesn't matter which one you use. / However, the first one is easier to work since since it gives you the answer directly. / Use the first equation as you move onto step 5 of the method for solving word problems. /*/

Frame: 2:32

$D = 7.3 - 2.7$

Voice:

Step 5 is to solve the equation and to check your work as you do so. Please solve this equation so that D is expressed as a single number.

Do this work in Box 9 on your work sheet. /**/

Frame: 2:33

$D = 7.3 - 2.7$

D = 4.6 MILES

Voice:

Your solution should be: D = 4.6 miles. // If you got some other answer, please check your subtraction. /#/

Frame: 2:34

JUDY"S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER WOULD SHE HAVE TO GO TO GET TO SCHOOL?

D = 4.6 MILES

Voice:

In the last step of our method, step 6, you should check to see if the answer is a reasonable one. // Since the total distance from Judy's house to school was 7.3 miles and she rode part of the distance, our solution should be a number less than 7.3. / Is it? / / Sure, it is. / Now, one way to check exactly is to add the number of miles Judy traveled on her bicycle and the distance she still has to go. / These should add up to the total distance from her house to school. /*/

Frame: 2:35

JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE RODE 2.7 MILES ON HER VICYCLE, HOW MUCH FARTHER WOULD SHE HAVE TO GO TO GET TO SCHOOL?

2.7 + 4.6 =

Voice:

Do this addition in Box 10 on your work sheet. /**/

Frame: 2:36

JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER WOULD SHE HAVE TO GO TO GET TO SCHOOL?

2.7 + 4.6 = 7.3

Voice:

2.7 miles plus 4.6 miles <u>does</u> add up to 7.3 miles or the distance from Judy's house to her school. / This means that we have solved the word problem and can be confident that our solution is correct. /*/

APPENDIX B

INSTRUCTIONAL SEQUENCE FOR SOLVING A PROBLEM

USING THE DAHMUS METHOD

Frame: 2:41

JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

Before we begin this problem let's reflect a moment. Recall that obtaining an answer is not your main goal since these problems are designed so that the answers are not difficult to obtain. Your main goal is to develop a method of translating the problem to mathematical symbolism and then working the problem using the mathematical symbols. We will approach the solution by translating the problem in parts. Translate <u>only</u> the part that is underlined when told to do so. Let's read the problem together. (Read the problem.) /*/

Frame: 2:42

JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

Notice when reading this problem, only one person was mentioned throughout the problem. Now, translate the underlined phrase in the first sentence. Write your translation in Box 10 on your work sheet. Raise your hand when you have finished. /**/

Frame: 2:43

J = 

JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE

RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER

WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

When translating the first pharse "Judy's school is," we can choose either J or S as the variable representation since only one person was mentioned in the problem. Let's choose J. Then the phrase is translated as "J equals." If you chose S, your answer is just as correct as the one indicated, but we will use J in the remaining explanation of this problem. /*/

Frame: 2:44

J = 7.3
JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE

RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER

WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

The next phrase is "7.3 miles from her house." The only part of this phrase requiring a translation is the number of miles, 7.3. None of the other words are key words. The first phrase to be translated in the second sentence is /*/

Frame: 2:45

J = 7.3

JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

"if she rode" tells us that she will ride part of the distance to school. Let's choose R to represent the distance ridden. /*/

Frame: 1-46

$\quad\quad\quad = 7.3$

JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE

R

RODE <u>2.7 MILES ON HER BICYCLE</u>, HOW MUCH FARTHER

WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

The translation is the variable R. Our next phrase requiring translation is "2.7 miles on her bicycle." Translate this phrase and indicate your answer in Box 11. Raise your hand when you have finished. /**/

Frame: 2:47

J = 7.3
JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE

R = 2.7
RODE <u>2.7 MILES ON HER BICYCLE</u>, HOW MUCH FARTHER

WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

This phrase yields only one translatable fact; the value 2.7 representing the number of miles. Notice here that the variable R just chosen is concerned with the distance traveled on the bicycle. Notice also there is an implied equality between R, the distance rode and 2.7, representing that distance in miles. The implication is - the distance ridden is 2.7 miles and the translation thus becomes equals 2.7. /*/

Frame: 2:48

J = 7.3
JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE

R = 2.7
RODE 2.7 MILES ON HER BICYCLE, <u>HOW MUCH FARTHER</u>

WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

"How much farther" is the next phrase and suggests we are interested in finding the remaining distance. Let us choose F to represent "how much farther." /*/

Frame: 2:49

J = 7.3
JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE

R = 2.7 F = ?
RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER

WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

Our last phrase provides us with the key connection between the variables. The phrase "would she have to go to get to school" reminds us that /*/

Frame: 2:50

    J               = 7.3
JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE

  R = 2.7                                    F = ?
RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER

       J  =  R + F
WOULD SHE HAVE TO GO TO GET TO SCHOOL?

Voice:

the distance to school, J, is the sum of the distance she rode, R, plus the remaining distance, F. The translation is now complete and yields the following information: /*/

Frame: 2:51

J = 7.3
JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE

R = 2.7 F = ?
RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER

J = R + F
WOULD SHE HAVE TO GO TO GET TO SCHOOL?

J = 7.3

R = 2.7

F = ?

J = R + F

Voice:

First, J equals 7.3. Second, R equals 2.7. Third, F equals some unknown value (a question mark). Fourth, and finally, J equals R plus F. We now need to solve the equations and find the value of F. Since we know the values of S and R, replace R and S in the last equation by the known values. Write your answer in Box 12. Please raise your hand when you have finished. /**/

Frame: 2:52

```
  J               = 7.3
JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE.  IF SHE

 R = 2.7                                F = ?
RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER

      J  = R + F
WOULD SHE HAVE TO GO TO GET TO SCHOOL?
```

J = 7.3

R = 2.7

F = ?

J = R + F

7.3 = 2.7 + F

Voice:

The result of substitution 7.3 for J and 2.7 for R gives us the equation, 7.3 equals 2.7 plus F. Now, solve the equation and write your answer in Box 13. Raise your hand when you have finished. /**/

Frame: 2:53

$$7.3 = 2.7 + F$$

$$7.3 - 2.7 = 2.7 - 2.7 + F$$

$$7.3 - 2.7 = F$$

$$4.6 = F$$

Voice:

7.3 equals 2.7 plus F. You may proceed by first subtracting 2.7 from each side of the equation and obtain 7.3 minus 2.7 equals 2.7 minus 2.7, plus F. This reduces to the third equation, 7.3 minus 2.7 equals F. Subtracting 2.7 from 7.3 we obtain 4.6 equals F. The value of F being 4.6 tells us it is 4.6 miles farther to school. Check your work with the work appearing on the screen. /3 sec./

Is 4.6 the correct answer? Let's check this value. /*/

Frame: 2:54

J = 7.3
JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE

R = 2.7 F = ?
RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER

J = R + F
WOULD SHE HAVE TO GO TO GET TO SCHOOL?

J = 7.3

R = 2.7 ANSWER: F = 4.6

F = ?

CHECK:

J = R + F J = R + F

Voice:

We will substitute one supposed answer into the equation J equals R plus F. Substituting /*/

Frame: 2:55

$J = 7.3$
JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE. IF SHE

$R = 2.7$ $F = ?$
RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER

$J = R + F$
WOULD SHE HAVE TO GO TO GET TO SCHOOL?

$J = 7.3$

$R = 2.7$

$F = ?$

$J = R + F$

ANSWER: $F = 4.6$

CHECK:

$J = R + F$

$7.3 = 2.7 + 4.6$

Voice:

we obtain the equation 7.3 equals 2.7 plus 4.6 Since 2.7 plus 4.6 equals 7.3 /*/

Frame: 2:56

```
  J           = 7.3
JUDY'S SCHOOL IS 7.3 MILES FROM HER HOUSE.  IF SHE

 R = 2.7                                 F = ?
RODE 2.7 MILES ON HER BICYCLE, HOW MUCH FARTHER

      J  = R + F
WOULD SHE HAVE TO GO TO GET TO SCHOOL?
```

Voice:

We see that the check yields a true statement, thus the supposed answer F equals 4.6 is indeed a solution to the problem. /*/

APPENDIX C

POSTTEST

POSTTEST

Name ____________________

DIRECTIONS:

For each problem write the equation or equations used to solve the problem in Box A. Use Box B to find the solution to the word problem and show your work.

Remember to check your solutions.

1. The length of a rectangle is 5 inches longer than its width. Its perimeter is 42 inches. What are the dimensions of the rectangle? (Hint: perimeter is the distance around an object.)

| |
|---|
| Box A |
| Box B |

2. The sum of two whole numbers is 74. One of the numbers is 28 greater than the other. What are the numbers?

| |
|---|
| Box A |
| Box B |

3. In a large office building, $\frac{3}{4}$ of the employees are women. How many of the 580 employees are women?

| Box A |
|---|
| Box B |

4. A student receives grades of 86, 75, and 80 on three tests. What must be his grade on a fourth test if his average on the four tests is to be at least 82?

| Box A |
|---|
| Box B |

5. Two cars start traveling from the same point in opposite directions. One car averages 35 mph., and the other 47 mph. How far apart are the two cars after 6 hours?

| Box A |
|---|
| Box B |

6. Two hundred comic books were distributed among three students. Jim received 42 books, and the remaining number was divided equally between Joe and Sally. How many books did Joe receive?

| Box A |
|---|
| Box B |

7. Tickets costing two dollars each were sold for a benefit play. If the total receipts were $374, how many tickets were sold?

| Box A |
|---|
| Box B |

8. A garden is 20 ft. wide and 30 ft. long. It has a circular cement fountain in it that is 5 ft. in radius. What is the area of planting space in the garden? (Hint: If the radius of a circle is r, then its area equals $\frac{22}{7}$ times radius times radius.)

| Box A |
|---|
| Box B |

9. One-third of Miss Jones' class was dismissed to go on a biology field trip. This means that 28 students were not dismissed. How many students are there in Miss Jones' class?

| Box A |
|---|
| Box B |

10. The perimeter of a square is 36 inches. A rectangle whose width is equal to the length of a side of the square has a perimeter of 42 inches. Find the width and the length of the rectangle. (Hint: perimeter is the distance around an object.)

| Box A |
|---|
| Box B |

APPENDIX D

RETENTION TEST

RETENTION TEST

Name ______________________________

DIRECTIONS:

For each problem write the equation or equations used to solve the problem in Box A. Use Box B to find the solution to the word problem and show your work.

Remember to check your solutions.

1. The width of a rectangle is 12 longer than its length. Its perimeter is 36 inches. What are the dimensions of the rectangle? (Hint: perimeter is the distance around an object.)

| Box A |
|---|
| Box B |

2. The sum of two numbers is 41. One number is 14 more than the other. What are the numbers?

| Box A |
|---|
| Box B |

3. At a large picnic, $\frac{1}{3}$ of the people are female. How many of the 510 people attending the picnic are female?

| Box A |
|---|
| Box B |

4. A student receives grades of 57, 63, and 72 on three tests. What must his grade be on a fourth test if his average on the four tests is to be 65?

| Box A |
|---|
| Box B |

5. Two trains start traveling from the same point in opposite directions. One train averages 35 mph., and the other 23 mph. How far apart are the two trains after 4 hours?

| Box A |
| --- |
| Box B |

6. A total of 250 candy bars were distributed among three students. Box received 116 candy bars, and the remaining number was distributed equally between Helen and Elizabeth. How many candy bars did Helen receive?

| Box A |
| --- |
| Box B |

7. Tickets costing 50¢ each were sold for a play. If the total receipts were \$255, how many tickets were sold?

| |
|---|
| Box A |
| Box B |

8. A garden is 15 ft. wide and 30 ft. long. It has a circular cement foundation in it that is 5 ft. in radius. What is the area of planting space in the garden? (Hint: If the radius of a circle is r, then its area equals $\frac{22}{7}$ times radius times radius.)

| |
|---|
| Box A |
| Box B |

9. Two-thirds of Miss Jones' class was dismissed from school for a field trip. This means 14 students were not dismissed. How many students are in Miss Jones' class?

Box A

Box B

10. The perimeter of a square is 24 inches. A rectangle whose length is equal to the length of a side of the square has a perimeter of 48 inches. Find the width and length of the rectangle. (Hint: perimeter is the distance around an object.)

Box A

Box B